SOME PROPERTIES OF STRONGLY NONEQUILIBRIUM FLOWS WITH POPULATION INVERSION IN SHOCK WAVES

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The effect of incident flow parameters and composition of a $CO_2 + N_2 + H_2O$(He) mixture on population inversion in the relaxation zone of a normal shock wave is considered.

Employment of rapid heating of a gas to obtain a population inversion between different quantum levels was proposed in [1]. In [2], using numerical integration of the equations of a normal discontinuity, an inversion was obtained between the levels 20°0–00°1 and 04°0–00°1 of CO_2 molecules in a mixture with N_2 and He, while in [3, 4] analytic solutions for determination of the number of particles at different oscillatory levels in the relaxation zone behind the shock wave front and in the entropy layer in flow around a wedge were obtained. These solutions are useful in analysis of the flow of multiatomic gases and gas mixtures whose nonequilibrium properties can be described by multitemperature relaxation theory.

Study of the structure of the relaxation zone behind the shock-wave front is important for the purpose of examining the general properties of nonequilibrium flow and for development of a possible laser medium. In the latter case it is important to know the effect of various gasdynamic parameters on the inversion level and amplification in the shock wave, the optimal concentration and composition of the laser mixture, the effect of a normal or oblique shock wave on particle distribution over levels in a strongly nonequilibrium flow, for example, in flow from a gasdynamic laser nozzle, etc.

For solution of these problems we will utilize a model of a nonequilibrium multiatomic gas with several relaxation levels [3], together with the condition that in the case of oscillatory relaxation the total oscillatory energy $\sum_i E_i$ is small in comparison to the total enthalpy h_0, i.e., max $\sum_i E_i/h_0 = \varepsilon \ll 1$. Then the solution of the gasdynamic equations may be sought by the method of successive approximations in the parameter ε. These solutions in explicit form define all macroscopic gas parameters behind the shock-wave front, and so the particle distribution over quantum levels can also be obtained from known relationships. We will apply the solutions of [4] to analysis of flow of typical laser mixtures $CO_2 + N_2 + H_2O$(He). The relaxation time τ_i for every oscillatory mode may be calculated from relationships presented in [5], using reaction rate data from [6] ($i = 1, 2, 3$ correspond to symmetric, deformation, and antisymmetric CO_2 molecule oscillation modes). As more accurate data on elementary process rates appear the relaxation time τ_i values can be refined.

We will consider the effect of incident flow conditions on population inversion in the relaxation zone behind a normal shock-wave front. Calculations show that the inversion curves have a typical resonance form with maximum inversion within the relaxation zone. Table 1 presents maximum inversion values $\Delta N_{max}/N$ in the relaxation zone for a mixture of 90% N_2 + 10% CO_2 for various M_∞ values, $p_\infty = 0.5$ atm, $T_\infty = 300$°K, together with the coordinates x_{max} at which this value is attained. It is evident that with increase in M_∞ the length of the inversion zone Δx is reduced, and x_{max} approaches the discontinuity front. Then with growth in M_∞ the inversion first increases, then falls, which is connected with the gradual population of higher quantum levels. We note that in view of bipolar kinetics behind the shock-wave front the relationship $p_0 x = \mathrm{const}$ is valid, so that with fixed values of gas velocity u_∞ and temperature T_∞ the length

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TABLE 1

M_∞	3	4	5	6	7	8
$\max \frac{\Delta N04^\circ 0}{N} 10^3$	2,46	7,0	9,5	10,0	8,5	7,4
$x_{max} 04^\circ 0$	0,75	0,35	0,16	0,07	0,04	0,02
$\max \frac{\Delta N20^\circ 0}{N} 10^3$	1,95	6,15	8,75	9,9	9,3	8,6
$x_{max} 20^\circ 0$	0,7	0,27	0,15	0,05	0,025	0,015
Δx_{mv}, mm $04^\circ 0$	7,95	2,9	1,65	0,82	0,45	0,27
Δx_{mv}, mm $20^\circ 0$	5,95	2,3	1,26	0,68	0,39	0,23

TABLE 2

x_{CO_2}	x_{N_2}	x_{H_2O}	x_{He}	$\max \frac{\Delta N04^\circ 0}{N} 10^3$	$\max \frac{\Delta N20^\circ 0}{N} 10^3$	x_{max} $04^\circ 0$, mm	x_{max} $20^\circ 0$, mm	Δx $04^\circ 0$, mm	Δx $20^\circ 0$, mm
0,1	0,9	0	0	9,5	8,75	0,16	0,15	1,65	1,26
0,1	0,89	0,01	0	9,15	8,55	0,15	0,12	1,4	1,14
0,1	0,8	0,1	0	6,8	6,4	0,13	0,09	0,67	0,54
0,1	0,8	0	0,1	7,9	7,2	0,2	0,15	1,5	1,2
0,1	0,5	0	0,4	4,4	4,25	0,28	0,2	0,9	0,74
0,1	0,1	0	0,8	−3,1	−2,9	0,25	0,23	0	0

Note: $M_\infty = 5$; $T_\infty = 300$°K; $P_\infty = 0.05$ atm.

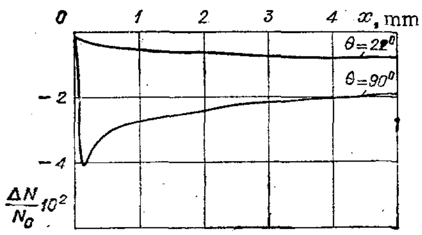

Fig. 1

of the inversion zone may be regulated by changing the pressure p, while not affecting the value $\Delta N_{max}/N$ (the indices ∞, 0 correspond to gasdynamic parameters ahead of and behind the shock-wave front).

Studies of the flow of expanding laser mixtures $CO_2 + N_2 + H_2O(He)$ in nozzles show that for fixed CO_2 concentration addition of water vapor or helium affects the population inversion favorably. Since in shock waves the CO_2 molecule 00°1 level is the lower laser level, the role of nitrogen must be that of collecting oscillatory energy from the antisymmetric oscillation mode of CO_2 [7]. However, in contrast to expansion flows, the population inversion zone is located in the high-temperature region. Hence the usefulness of water vapor or helium addition under these conditions is not obvious. Calculations show that for an unchanged quantity of CO_2 increase in H_2O or He concentration leads to a decrease in inversion and reduction in inversion zone length Δx behind the shock-wave front (Table 2).

We will consider effects related to development of a compression discontinuity in a nonequilibrium flow, for example, in flow of a mixture in a gasdynamic laser nozzle in the free flow mode. In this case the solution of [4] is also applicable, but because of the significant nonequilibrium in the external flow it is convenient to take the small parameter ε in the form proposed in [8]. It is obvious that the presence of a normal or inclined discontinuity will adversely affect the population inversion between 00°1−10°0 levels of the CO_2 molecules, the normal laser transition. Quantitatively, this effect will depend on the amount of heating of translational and oscillatory degrees of freedom in the discontinuity front. Oscillatory degrees of freedom directly behind the discontinuity front remain frozen. Depending on the intensity of the shock wave, several characteristic flow regimes may occur in the relaxation zone. If the gas temperature T_0 behind the front exceeds the antisymmetric mode temperature T_3 or is less than it within the limits $T_3/T_0 \lessapprox \Theta_3/\Theta_1$ (where Θ_i is the characteristic oscillatory temperature) the inversion behind the shock wave (if it took place directly ahead of the front) disappears at a distance $l \sim u_0 \tau_{1,2}$ i.e., at lengths of the order of the relaxation length of the symmetric and deformation modes of oscillation.

For $T_3/T_0 \gtrsim \Theta_3/\Theta_1$ and $T_0 > T_1$, $T_0 > T_2$ the inversion decreases but will be maintained over a longer distance, determined by the relaxation time $\tau_3(\rho_0, T_0)$. Finally, behind very weak shock waves the inversion change will take place in practically the same fashion as in front of the wave, but relaxation processes will be accelerated due to some increase in ρ and T. The above facts are illustrated by the data in Fig. 1, where the change in distribution of particles between the levels $00^{\circ}1-10^{\circ}0$ of the CO_2 molecule in the relaxation zone behind a normal compression discontinuity ($T_0 = 1950°K$) and a discontinuity inclined at an angle $\theta = 22°$ toward the incident flow ($T_0 = 600°K$) are shown.

Initial data were as follows: $M_\infty = 4.95$; $T_\infty = 397°K$; $T_{\infty 1} = 545°K$; $T_{\infty 2} = 553°K$; $T_{\infty 3} = 893°K$; $T_{\infty 4} = 918°K$; $p_\infty = 0.019$ atm and correspond to one variant of calculation of a gasdynamic laser nozzle with mixture 10% CO_2 + 89% N_2 + 1% H_2O at the output section performed by the authors of [9]. Although at this nozzle section population inversion has not yet been attained, the temperature inequality is significant. It is evident that the normal shock wave significantly decreases the value of $\Delta N/N$. The minimum in this curve is explained by the nonlinear distribution of particles among levels as a function of T.

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